



Rheological Study on Frequency Analysis of Rainfall Data of Lower River Ogun, South-Western Nigeria

Ayodele, A. ADEGBOLA¹; Adebisi, T. ADEYOKUNNU¹; and Olatunji, S. OLANIYAN²

¹Department of Civil Engineering, Ladoke Akintola University of Technology, Ogbomosho, P.M.B. 4000, Oyo State, Nigeria.

³Department of Civil Engineering, Osun State College of Technology, Esa-Oke, P.M.B 1011, Osun State, Nigeria.

* Corresponding author: adeyokunnuat20@gmail.com or bisonsoopy12@gmail.com

Abstract

Rainfall is the driving force for various engineering design such as hydraulic structures, road drainage system and bridges. The proper consideration of rainfall variability in time and space is the main challenge in hydrological design. The frequent occurrence of flood in Abeokuta has generated a lot of concern in Ogun State. These flood events have severe consequences on lives and properties. In this study, the rheological study on frequency analysis of rainfall data for Lower River Ogun was investigated. Ninety-eight (98) years rainfall data collected from 1920-2018 was obtained from Ogun-Oshun River Basin Authority Abeokuta, Nigeria. The rainfalls collected were analyzed statistically using the peak rainfalls. The peak rainfalls data were fitted to the three major statistical distributions namely normal, log-normal and log Pearson Type III model, while two plotting positions of Hazen and Weibull were used in determining the probabilities of exceedance with return period 25, 50 and 100 years respectively. The three models were evaluated statistically using Error analysis. The Weibull plot when matched with normal, log-normal and log Pearson Type III Distribution gave the highest coefficient of correlation of 0.959, 0.9377 and 0.9377 respectively. Hazen plot gave minimal error with Root Mean Square Error (RMSE) of 139.44, 151.85 and 151.85 for normal, log-normal and log-Pearson Type III distribution respectively. The Weibull plot matched with normal distribution gave the highest fit, most reliable and accurate predictions of the flood in the study area having the coefficient of determination $R^2 = 0.959$ and root mean square error RMSE 141.37 mm respectively.

Keywords

Probability of Exceedence; Rheological; Lower River Ogun; Plotting Position and Return Periods.

1.0 Introduction

Floods and their consequences all over the world are becoming too frequent threats to sustainable development in human settlement. The occurrences and reoccurrences of prolonged rainfall show that resultant floods all over the world in the recent times are becoming a source of concern to

researchers and government (Adeaja, 2008). In Nigeria alone, it is estimated that approximately 12% of the land area is within the 100 - year flood plain. However, the percentage of people dwelling in urban and rural areas within the flood plain is much higher (about 20%) (Oyegoke and Sojobi, 2012). Flood disasters have increased tremendously everywhere in Nigeria in recent times, due to the general rise in sea level globally. Baharudin and Bustan (2007) described rainfall data as the primary input parameter for various engineering design such as hydraulic structure, bridges and culverts. The details of rainfall and runoff analysis of each year is essential to estimate the relevant input value of rainfall-runoff relationship.

The Lower River Ogun is a major river running through the Abeokuta metropolis, which is the capital of Ogun state and eighth largest city in Nigeria. It is a major outlet for the sub-basins of the watershed. Abeokuta is highly urbanized and the land has been institutionalized for the development of residential, industries, recreational grounds and educational establishments (Olatunji, 2012). Due to Abeokuta's increasing population, land demand is an acute problem with residents using every available space along or on the bank of the river for personal use. Lower River Ogun is located in Abeokuta North local government of Ogun state with catchment area of about 9000 km². The river has a length of 480km. Ogun River has its origin in Oyo North as tributaries to Lower River Ogun. The communities ravaged by the flood include Eweko, and Obafemi/Owode local government areas in Ogun state (Eruola, *et al.*, 2012). At present, literature is limited on model for frequency analysis on Lower River Ogun, this calls for concerted effort to develop flood characteristics using Hazen and Weibull's plots.

Plotting Position in Estimating the Probability of Exceedance can be used to determine the occurrence of flood and its attendance damage to both life and property. In recent times especially in rural and urban dwellings and also in the agricultural establishments, floods have generated a lot of concern in Nigeria (Adeboye and Alatis, 2009). Adeboye and Alatis (2009) reported that hydrologists find it difficult to make accurate prediction of flood estimates using limited historic information of rainfall, runoffs and river water level. These can be attributed to lack of trained personnel and equipment for adequate assessment of these quantities on systematic basis. Tewolde and Smitthers (2008) described that in stochastic hydrology, the common and result oriented probability distributions in use are the normal, log-normal, log pearson type (III), gamma, and Weibull. The normal and the lognormal

distributions fit adequately to the peak rainfall and stream flow while for the extreme hydrologic variables, the Weibull and Gumbell distributions are used (Izinyon *et al.*, 2011). The uses of direct statistical and regional techniques in flood estimation have long been advocated for, but have witnessed little attention. The regional methods allow the design event to be estimated at ungauged site and the reliability of the estimated event increases due to increase in the incorporation of additional information (Ehiorobo and Nosa, 2014). However, the reliability of the predicted events for the design purposes at ungauged site is significantly lower when compared with the value at a gauged site (Izinyon *et al.*, 2011). Suchit *et al.* (2014) showed that the values of mean annual floods (MAF) at ungauged sites are less precise than the estimates at gauged site even with only one year of reliable data.

Adeboye and Alatise (2009) concluded that the peak discharge was plotted against their hydrologic years in order to determine variation of the exceedance of the discharge. The probabilities of exceedance of the discharges were determined using the seven plotting positions showed in **Table 1**. The frequency analysis could be used to estimate magnitudes of hydrological events and their corresponding return periods in order to predict the likelihood occurrence of high rainfall intensity which could lead to flooding. The objective of this study was to determine the flood characteristics of the study area by using three major statistical method and two plotting positions as outlined by previous researchers in the field of hydrology.

Table 1: Plotting positions used in determining the flood Estimation

Plotting Position	Formulae
Hazen	$P(Q \geq Q_i) = \frac{m - 0.5}{n}$
Weibull	$P(Q \geq Q_i) = \frac{m}{n + 1}$
Blom	$P(Q \geq Q_i) = \frac{m - 0.375}{n + 0.25}$
Cunnane	$P(Q \geq Q_i) = \frac{m - 0.4}{n + 0.2}$
California	$P(Q \geq Q_i) = \frac{m}{n}$
Gringorton	$P(Q \geq Q_i) = \frac{m - 0.44}{n + 0.12}$
Chegodajev	$P(Q \geq Q_i) = \frac{m - 0.3}{n + 0.4}$

(Source: Adeboye and Alatise, 2009)

From the **Table 1**

Q = anticipated streamflow, (m^3/s)

Q_t = streamflow of estimated return period to be exceeded (m^3/s)

m = rank order

n = number of observations.

The return periods of the anticipated discharges were determined by finding the reciprocal of the exceedance probabilities and is expressed by

$$T_r = \frac{1}{p} \quad 1$$

where:

T_r = return period

p = Probability of exceedance that is, the probability that a given flood is equal or exceeded.

2.0 Materials and Methods

2.1 Description of Study Area

The Lower River Ogun rises in Oyo state near Shaki, and flows through Ogun state into Lagos state. It lies between longitude $2^\circ 28' 33''$ and $3^\circ 48' 08''$ Easting and Latitude $6^\circ 37' 10''$ and $9^\circ 26' 39''$ Northing with catchment area of about 23,000 Km². Ogun River takes its source from Igaran hills at an elevation of about 530m above mean sea level and flows directly southwards over a distance of about 480 Km before it discharges into Lagos lagoon (Olatunji, 2012). Lower River Ogun is a big river cutting cross three states with more than twenty (20) tributaries, one of which is Oyan River. Its major tributaries are south of Abeokuta area (Oyegoke and Sojobi, 2012). River Onigbongbo and Ewekoro are tributaries to Lower River Ogun which lies within North of Abeokuta. It flows southward into Oyan River and supplies water to Abeokuta and its environments (Ikenweirwe *et al.*, 2007). The hydrological map of Lower River Ogun is presented in **Figure 1**.

2.2 Description of Study Area

The A 98 year (1920-2018) rainfall data of river Ogun was obtained from the Ogun-Osun River Basin Development Authority (OORBDA), Abeokuta, Nigeria. The annual rainfalls were ranked in descending order of magnitude based on the recommendations of the (United State of Water Resources Conservation USWRC, 1981). The probabilities of exceedance of the rainfall was determined using Hazen's and Weibull plotting position.

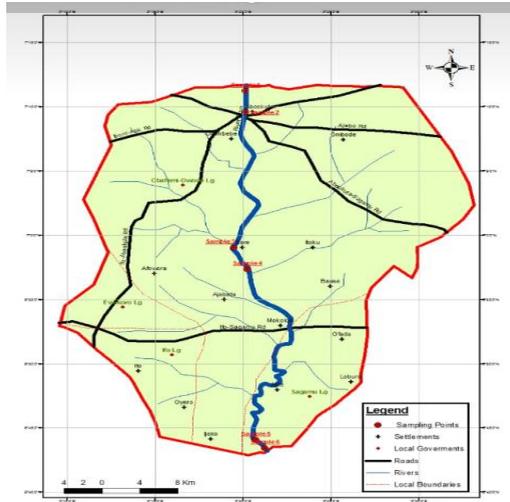


Figure 1: Hydrological Map of Ogun State showing Sampling Station

- i. The Hazen plotting position was used in the study and is represented by the following equation:

$$Q_T = \frac{M - 0.5}{n} \tag{2}$$

where:

Q_T is estimated return period to be equalled or exceeded

M is rank of rainfall data

n is number of years of study

The return period of anticipated rainfall will be determined by finding the reciprocal of the exceeded probability and is expressed by

$$T_r = \frac{1}{P} \tag{3}$$

where:

T_r = Return period

P = Probability of exceedence

- ii. The Weibull plotting position was also used and is represented by the following equation:

$$Q_T = \frac{m}{n + 1} \tag{4}$$

Q_T = estimated return period to be exceeded

M = rank of rainfall data

n = number of years of study.

The Hazen plotting position was outlined by (Hazen, 1914) and the Weibull plotting position was outlined by (Weibull, 1939).

2.2.1 Normal Distribution

For a symmetrically distributed data the most appropriate distribution variable is the normal distribution variable which is also called Gaussian distribution (Tilahun, 2006). The probability density function (PDF) of the distribution model according to (Chow *et al*, 1988) is given by

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-z^2/2} dz \tag{5}$$

where:

Z = standard normal variable, and

E = equivalent

The statistical parameters such as mean and standard deviation of the annual maximum rainfalls were determined using the method of moment, in which their respective equation are expressed as

$$Q = 1/n \sum_{i=1}^n Q_{max} \tag{6}$$

$$S_Q = \sqrt{\frac{\sum_{i=1}^n (Q_{max} - Q)^2}{n-1}} \tag{7}$$

Q = Mean of the Annual Maximum Rainfall (mm)

Q_{max} = Annual Maximum Rainfall (mm)

S_Q = Standard Deviation of Annual Rainfall (mm)

n = Number of years.

In this distribution, the intermediate variable will be determined using expression

$$W = \sqrt{\ln(1/p)} \quad 0 < p \leq 0.5 \tag{8}$$

where:

w is intermediate variable and

P is probability

The frequency factors corresponding to the return period of the ranked annual maximum rainfalls are determined using the expression

$$Z = W - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001388w^3} \quad 9$$

where:

P = probability of exceedance

W = Intermediate variable

Z = Standard normal variable

The predicted rainfall at various return period was determined using mathematical expression

$$Q_T = Q + Z S_Q \quad 10$$

Q_T , Q , Z and S_Q are previously defined

2.2.2 Log-normal Distribution

Large number of hydrological continuous variable and random variable tends to be symmetrical distributed. It is advantageous to transform the distribution to normal distribution by taking the logarithms of the annual maximum rainfalls (Tilahun, 2006).

The probability density function (PDF) under this distribution is given as:

$$F(x) = \frac{1}{x} \frac{\exp\left(-\frac{(\log x - \log \mu)^2}{2\sigma^2}\right)}{\sigma \sqrt{2\pi}} \quad 11$$

In this frequency analysis, the logarithms of the annual maximum rainfall was taken to base 10. The method of moments for the mean and the standard deviation of the ranked annual maximum rainfall were determined by the expression

$$y = \frac{1}{n} \sum_{i=1}^n \log Q_{max} \quad 12$$

$$S_y = \sqrt{\frac{\sum_{i=0}^n (y_i - y)^2}{n-1}} \quad 13$$

where:

Y = Mean of y (mm)

S_y = Standard Deviation (mm)

Y = $\log Q_{max}$ (mm) and n is number of years.

The intermediate variables and frequency factors corresponding to the ranked annual maximum rainfall was determined by equations 12 and 13 respectively. The statistical variate and predicted rainfall under this distribution were determined respectively by the expressions

$$y_T = y = Z. S_y \tag{14}$$

$$Q_T = 10^{(y + Z. S_y)} \tag{15}$$

where:

y_T = Variable of the annual maximum rainfall at return period T (years)

y =mean of the logarithms of the historic annual maximum rainfall

S_y , Q_T and z are as previously defined

2.2.3 Log Pearson Type III distribution

Large Log Pearson Type III distribution is referred to as the three parameter fit. Due to its performance in stochastic hydrology, it has been adopted in some countries as the standard distribution for rainfall frequency analysis (Sikorska, 2017). The probability model is given as

$$= \frac{\lambda^B (y-e)^{B-1} e^{\lambda y - e}}{x^r} \tag{16}$$

$$y = \log_x, \lambda = 5_{y_r}, B = 2 \left[\frac{2\epsilon = y = S_y B}{\sqrt{\frac{C_s(y)}{C_s(y)}}} \right] \frac{1}{\sqrt{B}} \tag{17}$$

Using the log Pearson Type (III) Distribution, the frequency factors corresponding to the predicted annual maximum rainfall was given by (Adeboye, 2005) as:

$$K_T = 2 + (2^2-1)K + 1/3(2^3-67)K^2 - (22-1)K^3 + 2K^4 + 1/3K^5 \tag{18}$$

where K_T = frequency factor

K = expressed as $C6/5$, and C = constant variable

Z = as previously defined

At each return period, the predicted rainfall were determined by

$$Q_T = 10^{(y + K_T. S_y)} \tag{19}$$

In order to compare, a model output to observe the data criteria for making such a comparison must be identified (Green and Stephenson, 1985).

Visual comparison of the plotted predicted and observed rainfall can be very useful in assessing the accuracy of the model output. Statistical procedures were used in the analysis for evaluating the performance of the distribution and coefficient of determination. Absolute differences between the predicted and observed rainfalls were determined using the equation as outlined by (Izinyon, 2011)

$$\text{RMSE} = \sqrt{n\varepsilon(P - O^2)} \quad 20$$

Where:

RMSE = root mean square error (mm)

P = predicted rainfall under each distribution (mm)

O = observed rainfall (mm)

n = number of occurrence

3.0 Results and Discussions

3.1 Hazen's Plotting Position

The The result of the probabilities distribution using Hazen's plotting position for normal, log-normal and log Pearson type (iii) distribution model is presented in **Table 2** and **Figure 2** respectively. The observed rainfalls in Figure 2a, b and c at return periods of 25, 50 and 100 years were 1577.65, 1600.24 and 1611.448 mm respectively and the coefficient of determination was 0.976, **Figure 2a** shows the graphical illustration of annual maximum rainfalls against the probability of non-exceedance for the normal distribution using Hazen's plotting position. From this figure, it was observed that at the return periods of 25, 50 and 100 years, the predicted rainfalls were 1348.20, 1364.82 and 1373.04 mm respectively with the coefficient of determination R^2 was 0.9465.

The computations of predicted rainfalls are lower compared with the observed rainfalls. The minimum absolute differences between the predicted and observed rainfalls were 229.45, 225.42 and 238.44 mm respectively. It was observed that **Figure 2b** exhibits similar graphical pattern as **Figure 2a** but under the log-normal distribution. At return period 25, 50 and 100 years, the predicted rainfalls were 1340.34, 1356.52 and 1364.69 mm respectively with R^2 0.9199. The estimation of predicted rainfalls are lower and do not compare well with the observed rainfalls. The RMSE 151.85 mm was the highest under log-normal plotting position. **Figure 2c** present the graphical illustration under the log Pearson Type III distribution. At return periods of 25, 50 and 100 years, the predicted rainfalls were 1340.34, 1356.52 and 1364.69 mm respectively and R^2 value was 0.9199 which is similar to that of log normal distribution. The normal distribution has the highest degree of correlation for the predicted rainfalls. The

highest absolute difference between the observed and predicted rainfalls were obtained under this distribution. The findings were in contrast with the work of Adeboye and Alatisé (2007) which reported that normal distribution combines with Weibull's formula gave the best fit. This model could predict likelihood occurrence of flood with the return period 25, 50 and 100 years respectively.

3.2 Weibulls Plotting Position

The The results of the probabilities distribution of normal, log normal and log Pearson type (iii) distribution model using Weibull Plotting Position is presented in **Table 2** and **Figure 3** respectively. The observed rainfalls in **Figure 3a, b** and **c** were 1577.99, 1600.24 and 1611.48 mm with R^2 value 0.976 at return periods of 25, 50 and 100 years respectively. In **Figure 3a**, at the return periods of 25, 50 and 100 years, the predicted rainfalls for the normal distribution using Weibull's Plotting Position were 1342.20, 1358.40 and 1366.58 mm respectively and the coefficient of determination R^2 is 0.9599. The root mean square error (RMSE) was 141.37 mm. The computation of predicted rainfalls compared well with observed rainfalls. The result for log normal distribution model is presented in **Figure 3b**. It was observed that the predicted rainfall were 1331.95, 1348.03 and 1356.14 mm respectively with coefficient of determination R^2 value 0.9377. The root mean square error value is 154.34 mm with the absolute difference of 246.05, 252.21 and 255.34 mm respectively. **Figure 3c** is the graphical representation of log-Pearson Type III distribution model. The predicted rainfalls were 1331.95, 1348.03, and 1356.14 mm respectively at the same return periods with the coefficient of determination R^2 value 0.9377. The root mean square error for this distribution is 154.38 mm while the absolute difference between the observed and predicted rainfalls at return period of 25, 50 and 100 years were 246.05, 252.21 and 255.34 mm respectively. The normal distribution model gave the highest fit using Weibull Plotting Position.

The comparison between predicted rainfalls and return period for different probability distribution model is presented in **Figure 4**. The highest predicted rainfall was observed in normal probability distribution model at 100 year return period using Hazen Plotting Position while the highest coefficient of determination R^2 value 0.959 was also observed in normal distribution model when using Weibulls Plotting Position. The normal distribution deviated greatly from log-normal and log Pearson Type III distribution for different plots under consideration. The observed deviation of normal distribution may

be attributed to the failure of not transforming the rainfall data on logarithmic scale.

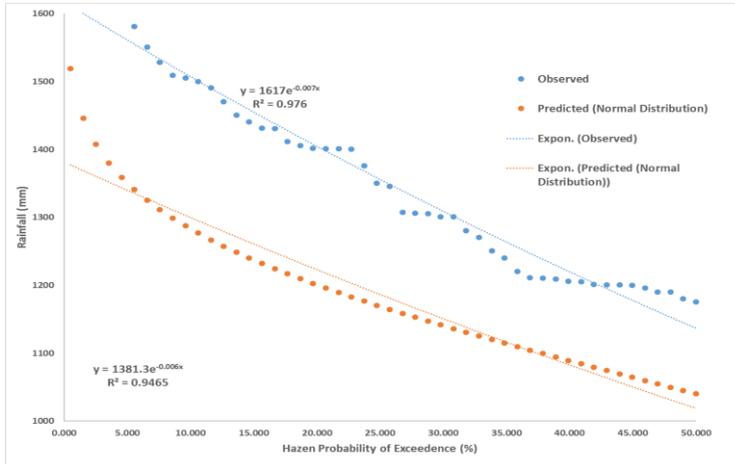


Figure 2a: Normal Distribution using Hazen's Plotting Position

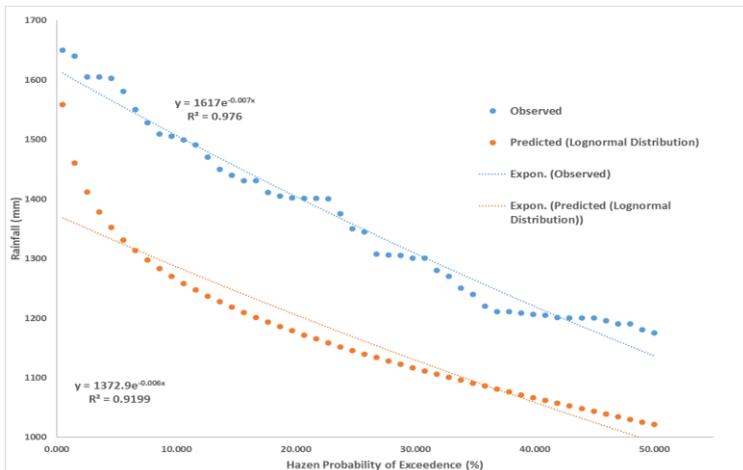


Figure 2b: Log-Normal Distribution using Hazen's Plotting Position

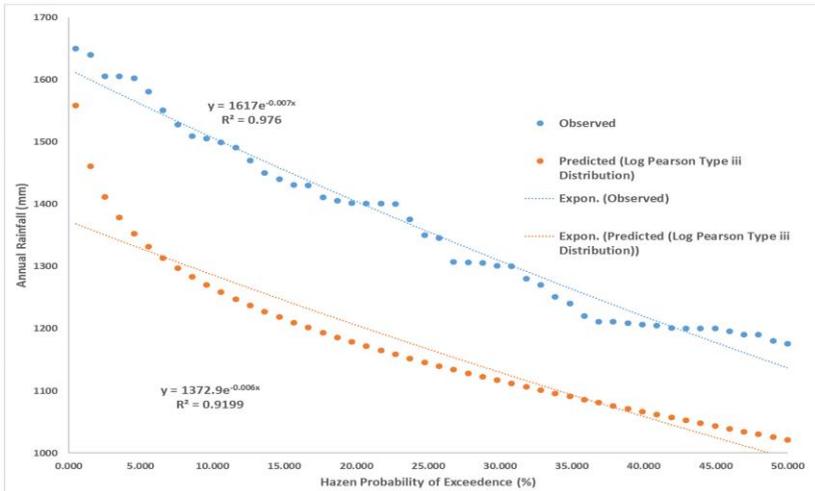


Figure 2c: Log Pearson type (iii) Distribution using Hazen's Plotting Position

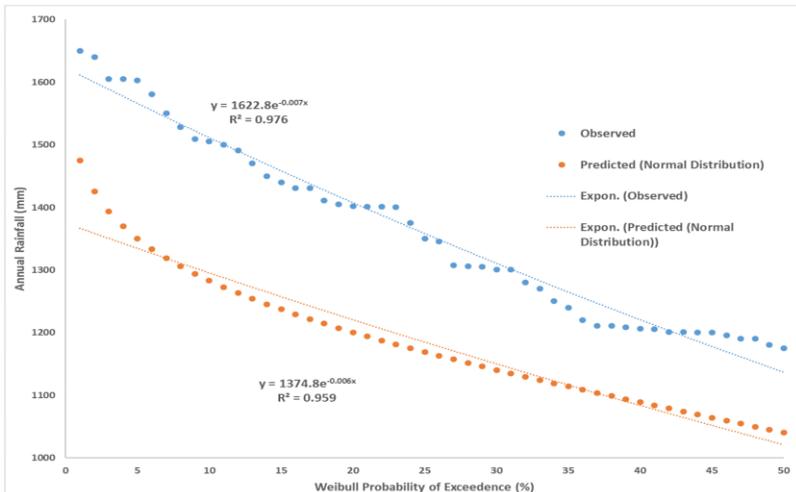


Figure 3a: Normal Distribution using Weibull's Plotting Position

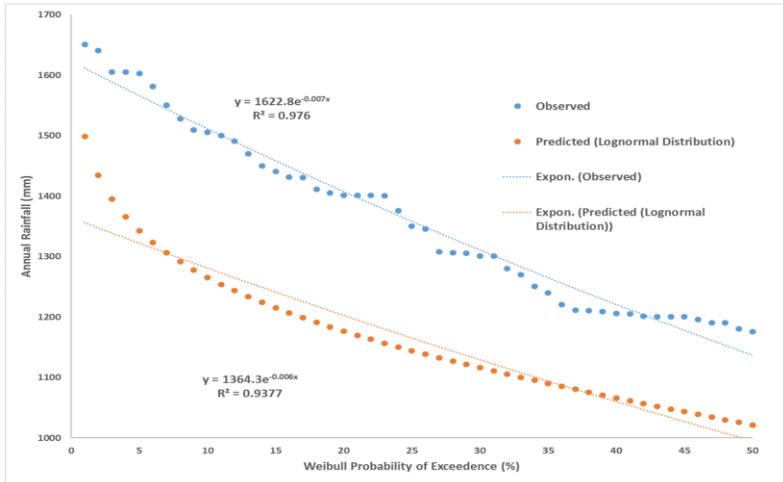


Figure 3b: Log-Normal Distribution using Weibull's Plotting Position

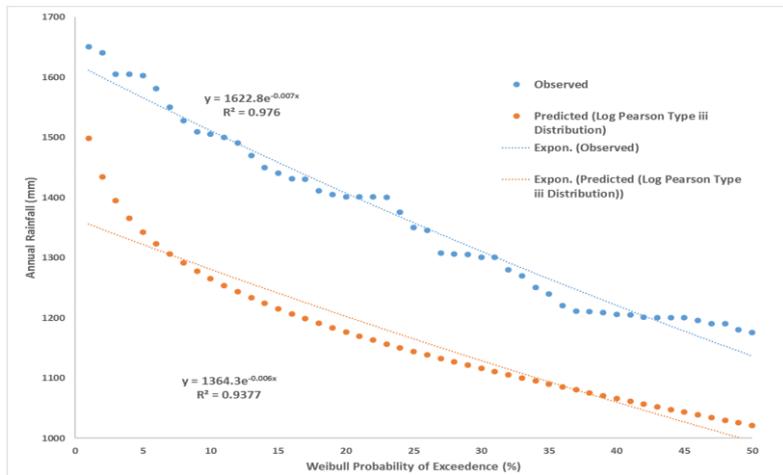


Figure 3c: Log Pearson type (iii) Distribution using Weibull's Plotting Position

Table 2: Coefficient of Determination, Root Mean Square and Absolute Difference between Observed and Predicted Rainfall

	Plotting Position	Probability			Distribution			log-Pearson type (iii)		
		Normal			Log normal			log-Pearson type (iii)		
Regression Coefficient (R ²)	Hazen	0.9465			0.9199			0.9199		
	Weibull	0.9599			0.9377			0.9377		
Root Mean Square Errors (RMSE) (mm)	Hazen	139.44			151.8523			151.8523		
	Weibull	141.37			154.3828			154.3823		
Return Period (T)		25	50	100	25	50	100	25	50	100
Predicted Rainfall (mm)	Hazen	1348.20	1364.82	1373.04	1340.34	1356.52	1364.69	1340.34	1356.52	1354.69
	Weibull	1342.20	1358.40	1366.58	1331.95	1348.03	1356.14	1331.95	1348.03	1356.14
Absolute Difference (mm)	Hazen	229.45	235.42	238.44	237.65	243.72	246.79	237.65	243.72	246.79
	Weibull	235.79	241.84	244.90	246.05	252.21	255.34	246.05	252.21	255.34

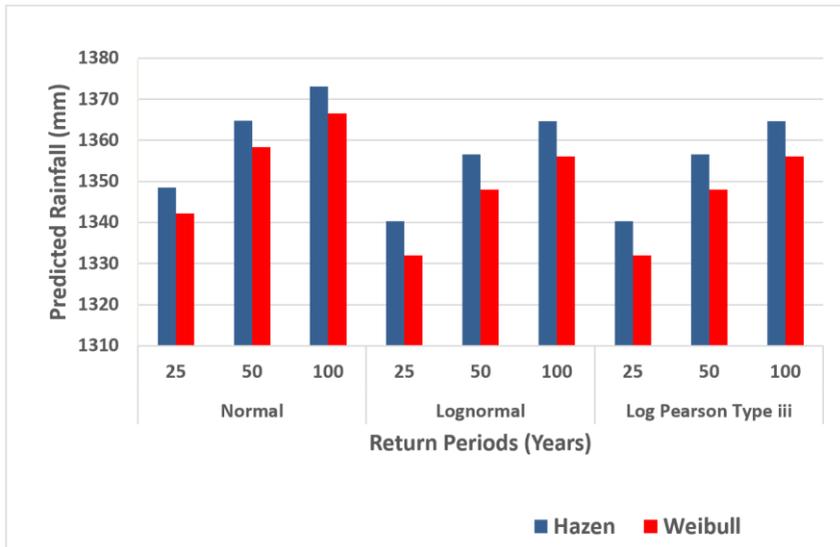


Figure 4: Comparison of Predicted Rainfall using Different Probability Distribution

4.0 Conclusion

The normal distribution had the highest coefficient of determination using Weibull Plotting Position, normal distribution model had minimum (RMSE) when matched with Weibull's Plotting Position. The minimum absolute differences at return periods of 25, 50 and 100 years were obtained under normal distribution. Flood frequency analysis shows that under normal distribution, Weibulls Plotting gives the best fit.

5.0 References

- Adeboye, O.B. and Alatise, M.O. (2009). Performance of Probability Distribution and Plotting Position in Estimating the Flood of River Osun at Apone Sub-basin, Nigeria. *Agricultural Engineering International: The EJournal manuscript* LW 07 007. 6(9):1-21.
- Adeaja, W.M. (2008). Vulnerability of Flood Risk Analysis. *Journal of Global Environmental Changes*; 16(3) 268-281.
- Baharudun, F. and Bustan, I. (2007). Determination of Rainfall-Runoff Characteristics in urban Areas: Sungal Kerayong catchment on Civil Engineering Research. *Global Journal Geological SG*. 2:111-123.
- Chow, V.T., Maidment, R.D., and Larry, W.M. (1988). *Applied Hydrology* International Edition, McGraw Hill Inc, Singapore, 350-407.
- Ehiorobo, J.O. and Nosa, U. (2014). Flood Frequency Analysis in the River Niger Basin at Onitsha Bridge Head. *Journal of Engineering Research, UNILAG*, 19(1): 33-44.
- Eruola, A.O., Ufeogbune, G.C., Oluwasanya, G.O., and Ede, V.A (2012). Effect of Climate Change on water Balance of Lower Ogun River Basin. *Special Publication of the Nigerian Association of Hydrological Sciences*, 44(4): 360 – 367.
- Eychaner, J.H (2015). *Lesson from a 500-Year Record of Flood Elevations*, Association of State Floodplain Managers, Technical Report 7, www.hec.usace.army.mil/hec/geo/hms Accessed 2015-06-27.
- Green, I.R, and Stephen, D. (1985). Comparison of Urban Drainage Models for use in South Africa, *Journal of Water Resouces Commission*, 11(5):76-86.
- Hazen, A. (1914). Storage to be provided in Impounding Reservoir for Municipal

Water Supply, *Transaction of ASCE*, 77:1547-1550.

- Ikenweirwe, B.P, Verma, P.K., and Joshi, B.L. (2017). The relationship between rainfall and discharge analysis of sewer popurs districts. *American Society of Civil Engineering*, 20(6):21-46.
- Izinyon, O.C, Ihimekpen, N. and Igbinoba G.E. (2011) *Flood Frequency Analysis of Ikpoba River Catchment at Benin City Using Log-Pearson Type III Distribution*, *Journal of Emerging Trands in Engineering and Applied Sciences (JETEAS)*, Vol. 2(1): Pp. 50-55.
- Olatunji, T. (2012). Towards Effective Hydrological Measurement downstream of Oyan Dam. *Special Publication of Nigerian Association of Hydrological Science*. 5(13): 121 - 131.
- Oyegoke, S.O, and Sojobi, A.O. (2012). Developing Appropriate Techniques to Alleviate the Ogun River Network Annual Flooding problems. *International Journal of Scientific and engineering research*, 3(2): 1-7.
- Sikorska, A.E. (2017). Calculating a Frequency Model in stage space to account for flood mitigation. "*Journal of Advance in Water Resources*", 105:51-66.
- Suchit, K.R., Sunil, K., Arvind, K.R. and Dara, R.P. (2014). Climate change variability and Rainfall Probability for Crop Planning in few Districts of Central Indian. *Climate SG (4)*:394-403.
- Tewelde, M.H. and Smitthers, J.C. (2008). The Flood Routing in Ungauged Catchment Using Muskingum Methods, *WRC, Water S.A* 32(3):164-180.
- Tilahun, K.(2006). The characterization of rainfall in the arid and semi arid regions of Ethiopia, *Water SA*, 32(3):429-439.
- USWRC (1981). *Guideline for ranking hydrological parameters*. Third Addendum to Fourth Edition Volume 2 recommendation.
- Weibull, W. (1939). *A Statistical Theory of Strength of Materials Ing. Vet A.K, Handl*, 151, Genelstabens Litografiska Anstals Foclg Stockholm, Sweden.