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Tilt Angle Optimization of a Flat Plate Solar Collector for Bauchi

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Abstract
Conventional flat-plate solar water heaters are primarily of the fixed types which often are inefficient due to seasonal fluctuation in solar radiation received. This research presents the optimization of tilt angles for flat plate collector for water heating for pre-primary education pupil's during school hours using Bauchi climatic condition. Design equations involving hour angle, latitude, declination, ratio of the average beam radiation and, characteristics properties of the proposed solar collector were coded in MATLAB programming language to forecast the categorization of the monthly averaged daily sunshine hour, optimal tilts, collector useful energy, and energy delivered, useful hot water energy and, dumped water energy. The meteorological data for Bauchi was used to facilitate and generate results. To enhanced total solar radiation estimate on the tilted collector surface, the monthly average index was based on a range covering all sky conditions of overcast, cloudy I, cloudy II, sunny I and, sunny II conditions. The results indicated an optimal tilt angle of 36° and 4° in the first and second quarter, while the third and fourth quarters were 12° and 42° respectively with annual tilt at 24°. The comparative annual useful hot water energy of the collector to the conventional tilt position for quarterly, annual and monthly tilt angles corresponds to 1%, 3% and 10% improvement. The loss in performance of a fixed horizontal collector compared to that of a collector with a conventional tilt is 7%. This improved technique will increase flat plate thermal collectors' performance, minimize system use of auxiliary energy and reduce the material cost for development. To validate this improved technique, further work is needed to predicts, developed and experiment a prototype flat plate solar water heater with the proposed orientations.

Keywords: *Flat plate collector; solar energy; optimal tilt angle; enhance thermal performance.*

1.0 Introduction

The availability of solar energy in most countries of the world has reduced its over-dependence on the use of conventional energy sources for applications where it could be easily harnessed (Babu and Chinnapandian, 2017). Flat plate collectors for water heating applications are mostly found in areas such as bathing, sterilization of medical equipment, hair dressing, district heating, washing e.t.c, where the temperature requirement does not exceed 80°C (Ternenge *et al.*, 2021). Despite these numerous applications and their ability to cut down energy costs in homes, offices, commercial centers, the non-attainment of their design temperature throughout the year for collectors with fixed tilt posed a limitation in their applications (Jyoti *et al.*, 2015). This constraint is due to the seasonal variation across the year which amounts to changes in the amount of solar radiation received by locations across the earth (Subba *et al.*, 2014). This inadequacy can be overcome either through the provision of an automatic sun tracker to the system or the adaptation of a monthly tilt angle. The former is a better option but could be constrained by its cost implication allowing the latter as an affordable option especially for low-cost solar water heaters. The amount of solar radiation received by solar collectors is a function of varying factors ranging from its orientation, tilt angle, the angular position of the sun at solar noon with concerning the plane of the equator and the sunrise hour angle, and the azimuth angle (Selfa, 2015). For instance, solar collectors with high solar declination are made to accept more direct sunlight leading to higher thermal performance, whilst those located in locations where the solar declination is low, the angle of sunlight striking the surface decreases and thus, experience reduction in the amount of solar radiation received (Subba *et al.*, 2014). Solar thermal devices integrated with sun trackers ensure that the day and seasonal solar paths are followed. They are known for harnessing maximum radiation which are more efficient and thus, requires fewer materials for development. However, those with fixed tilt often called stationary type of solar collectors have an aperture area that serves both for intercepting and for absorbing solar radiation (Emmanuel *et al.*, 2020).

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Several researches works for determining tilt angles are found in the literature. Jamil and Tiwari (2009) examined the theoretical aspects of choosing a tilt angle for Indian stations and found that the annual optimum tilt angle to be approximately equal to the latitude of the location, loss in the amount of collected energy around 1 % if the angle of tilt is adjusted seasonally and loss of energy when using the yearly average fixed angle was around 15 % compared with the monthly optimum tilt. Felix and Emmanuel (2013) using a mathematical model found the annual average fixed and seasonal tilt angle to be 26.8° in the West Region of Ghana with a loss of 1% and 0.6% respectively compared to the monthly optimum tilt. Hassane and Driss (2013) studied the optimal tilt angle for maximum solar radiation collection for sites in the Mediterranean region using regression analysis to develop quadratic and linear models using computer simulation. They found that the quadratic model gave a higher precision of the prediction compared to the linear model. Using a heuristic algorithm and particle swarm optimization, Feng-Jiao and Tian-Pau (2015) established that the optimal tilt angles are positive for most of the months of the year while negative for summer months from May to July in Taiwan under clear sky taking into consideration of various periods. Alsadi *et al.* (2016) utilized ASHRAE clear sky model in determining optimum tilt angles in locations of the earth. Results of the optimum tilt angles presented in a polynomial form in comparison with measured data and those of NASA, suggested these polynomials could be used especially in mid and high latitudes in the two hemispheres. The studies of Soulayman *et al.* (2016) revealed that changing the tilt angle on a monthly basis maintains approximately the total amount of solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value and achieves a yearly gain in solar radiation up to 1.8 times of the case of a horizontal surface. The results of the mathematical model based on the monthly average hourly measured values for diffuse and global solar radiation on a horizontal surface in the test field for Cairo, Egypt by Aiat (2019) indicated that for non-fixed systems, changing the tilt angle monthly, seasonally, two times per year and yearly could increase the collected solar energy over that of the optimal fixed tilt by 5.5%, 4.6%, and 4%, respectively. Isaac *et al.,* (2020) carried out a coding system for parametric optimization of absorber plate thickness, back insulation thickness and tilt angle of a flat plate collector using a written MATLAB program. For maximum solar collection for a plate in the geographical location of Zaria, the tilt angle was found 20° tilted from the horizontal facing the south. Hassanian *et al.* (2021) work show annual solar cumulative radiation energy gained via a monthly tilt angle to be approximately 7% higher than that with an annual tilt angle. In addition, the seasonal and bi-annual tilt angles have about 6% more annual cumulative radiation absorption than the annual tilt angle.

This research work seeks to optimize the tilt-angle for flat plate collectors on monthly, quarterly, and annual basis, and make a comparison to the conventional flat plate collectors' tilt for energy harnessing. The estimate for the total solar radiation on the tilted collector surface is deployed by an improved approach of utilizing five sky conditions namely overcast, cloudy I, cloudy II, sunny I and, sunny II. Previous works have failed This work will afford the choice of selecting among options, the most suitable orientation for achieving required temperature for hot water supply all year round with minimum auxiliary energy supply.

2.0 Methodology

2.1 Average daily radiation

For a horizontal surface, the average daily radiation, H is often computed by utilizing direct and diffuse irradiation, while the horizontal H_o daily extraterrestrial radiation on a given surface is obtain from

$$H_o = \frac{24\times3600}{\pi} G_{sc}\left[1 + 0.033\left(360\frac{n}{365}\right)\right] \times \left[\sin\varphi\sin\delta\left(\frac{2\pi\omega_s}{360}\right) + \cos\varphi\cos\delta\sin\omega_s\right] \quad (1)$$

$$\hat{N} = \frac{2}{15}\cos^{-1}(-\tan\varphi\tan\delta) \quad (2)$$

Where:

the hour angle ω_s, latitude ϕ and declination δ are related by:

$$\cos\omega_s = -\tan\phi\tan\delta \quad (3)$$

Where:

δ is the solar angle of declination and is approximately given as (Selfa, 2015):

$$\delta = 23.45\sin\left[\frac{360}{365}(284 + n)\right] \quad (4)$$

The value 1367 W/m^2 has been recommended for the solar constant, G_{Sc} (Abdul-Azeez, 2011).

2.2 Ratio of monthly radiation

The ratio of the monthly average daily radiation on a horizontal surface, H to the monthly average daily extraterrestrial radiation, H_o called the monthly average index, KT is expressed in Equation 5 as reported by Hassanian *et al.* (2021).

$$KT = H/H_o \quad (5)$$

This technique is limited by its consideration for only clear days. For an enhancement over the method, five categories of sky conditions namely overcast, cloudy I, cloudy II, sunny I, and sunny II are recommended and this includes coefficients of 0, 0.2, 0.4, 0.8, and 1 respectively.

2.3 Total solar radiation on tilted collector surface

The total solar radiation on the tilted collector surface, H_T for this research work is that proposed by Rasouli and Puig (2019).

$$H_T = H \times R = R \times KT \times H_O \tag{6}$$

Where:

R is ratio of the daily average radiation on a tilted surface to that on a horizontal surface for each month as defined by Equation 7.

$$R = \left(1 - \frac{H_d}{H}\right) \times R_b + \frac{H_d}{H}\left(\frac{1+\cos\beta}{2}\right) + \sigma\left(\frac{1-\cos\beta}{2}\right) \tag{7}$$

Where:

R_b is defined as the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each month as given in Equation 8.

$$\mathrm{R_b} = \frac{\cos(\emptyset+\beta)\cos\delta\sin\omega_s{}^1 + \left(\frac{\pi}{180}\right)\omega_s{}^1\sin(\emptyset+\beta)\sin\delta}{\cos\emptyset\cos\delta\sin\omega_s + \frac{\pi}{180}\omega_s\sin\emptyset\sin\delta} \tag{8}$$

Where:

$$\omega_s{}^1 = min(A\ or\ B) \tag{9}$$

$$A = \cos^{-1}(-\tan\emptyset\tan\delta) \tag{10}$$

$$B = \cos^{-1}(-\tan(\emptyset+\beta)\tan\delta) \tag{11}$$

and "min" means the smaller of the two values in the bracket.

2.4 Solution scheme of the study

The optimized tilt angles on monthly, quarterly, and annual basis are calculated for the values for which received radiation of the flat plate collector is a maximum. The study for the collector optimization was carried out using a program written in MATLAB. Design equations involving hour angle, latitude, declination, ratio of the average beam radiation and, characteristics properties of a proposed solar collector length of 1700mm, width of 1200mm, and an aluminum absorber plate of 4mm thickness, 50mm fiber-glass insulation material for bottom and edge heat loss were coded in MATLAB language. The meteorological data for Bauchi was used to facilitate and generate results for the study. The estimation for the optimal tilt was carried out from 0 degrees to 48 degrees in step increment of two degrees. The optimal tilt is the angle corresponding to the collector inclination for which the meteorological data of Bauchi gives maximal solar radiation on the collector surface. Also, equations were written to forecast the categorization of the monthly averaged daily sunshine hour for Bauchi, collector useful energy, and energy delivered, useful hot water energy and, dumped water energy for solar collector load water tank volume of 65 liters for the year.

3.0 Results and Discussions

The estimated monthly averaged daily radiation for Bauchi is predominantly of sunny II type, receiving more than 800W/m^2 throughout the year with exception in the months of June and July where it gets between 700 and 800 W/m^2 (figure 1). Thus, the state solar radiation is substantial for solar thermal systems load demand if properly design.

The optimum tilt angle for the months of the year displays a decreasing trend from 44° in January to 0° in July and thereafter increases to 46° in December (table 1). This improved technique provides a range of solar radiation conditions from overcast, cloudy I, cloudy II, sunny I, and to sunny II showcasing the months of October and April with peak values for the energy of delivered hot water for optimal monthly and conventional tilt with values of 64.2kWh/month and 61.7kWh/month respectively, and that of October is 64.1 kWh/month for quarterly tilt, and annual optimum angle estimated at 63.3 kWh/month all in October (figure 2). The total energy of the delivered hot water for the collector's surface is least for fixed horizontally collector with a value of 586.8 kWh/year, maximum for surface inclined at the optimal monthly tilt receiving 694.6kWh/year, while the optimal annual and collector tilt latitude of Bauchi had 655 and 634 kWh/year respectively, and quarterly to be 639.5kWh/year (figure 2). The month of January had overall peak useful hot water energy of 44.8 kWh/month for collector set at optimal monthly tilt whilst the system least useful hot water energy delivery of 14 kWh/month in July (figure 3). However, the results revealed that the months of June to August depicted useful hot water energy below 20kWh/month. Evaluating the pair of energy used in meeting the system load and hot water load demand suggests least amount of auxiliary energy will be needed on annual basis when the collector inclination is set on the optimal monthly angle, with progressive increment for quarterly, annual, location latitude and a horizontal fixed collector (table 2). The conventional tilt, quarterly, annual, and monthly optimum tilt had an increasing trend for system load excess energy with horizontal fixed tilt exhibiting the least values signifying the indisputable fact of utilising monthly optimal tilt for stationary flat plate collectors (figure 4). In comparison to a collector inclined to the location latitude of Bauchi (conventional tilt), there is an increment percentage improvement of 1, 3, and 10% for quarterly, annual, and monthly optimal angles and a negative 7% for a collector fixed horizontally (table 2). This result is in agreement with the earlier work of Bala *et al.* (2019) who obtained improvement of 7.3% for radiation gain for a stationary collector fixed at an angle throughout the year in Kano, while Abhishek *et al.* (2012) reported a 4.56% for India. Reza *et al.*, (2021) presented predictable annual solar cumulative energy gained through a monthly tilt angle to be roughly 7% higher than that attained for an annual tilt angle whereas the seasonal and bi-annual tilt angles have approximately 6% more annual cumulative energy absorption than the annual tilt angle. As envisaged, there is potential in meeting system load demand throughout the year with the adoption of monthly tilt angle for fixed flat collectors as against the conventional tilt usage of the location latitude.

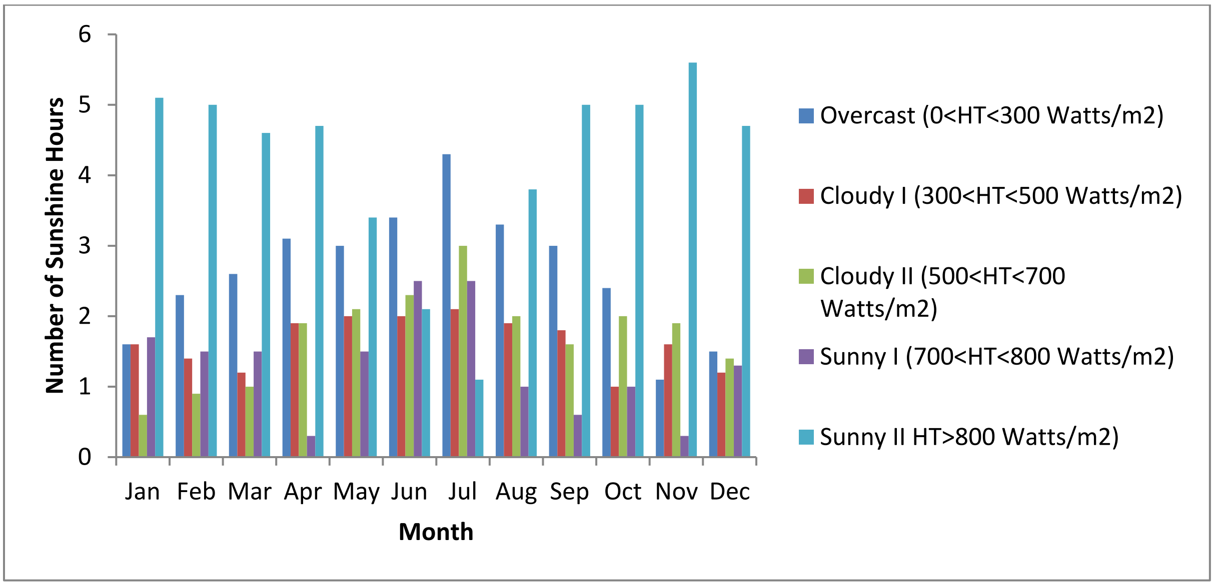

Figure 1: Categorization of monthly averaged daily sunshine hour for Bauchi

Table 1: Optimal tilt angle

Month	**β_{opt} [°] monthly**	**β_{opt} [°] quarterly**	**β_{opt} [°] annually**
January	44	34	24
February	38		
March	26		
April	14	4	
May	2		
June	2		
July	0	12	
August	10		
September	18		
October	30	42	
November	42		
December	46		

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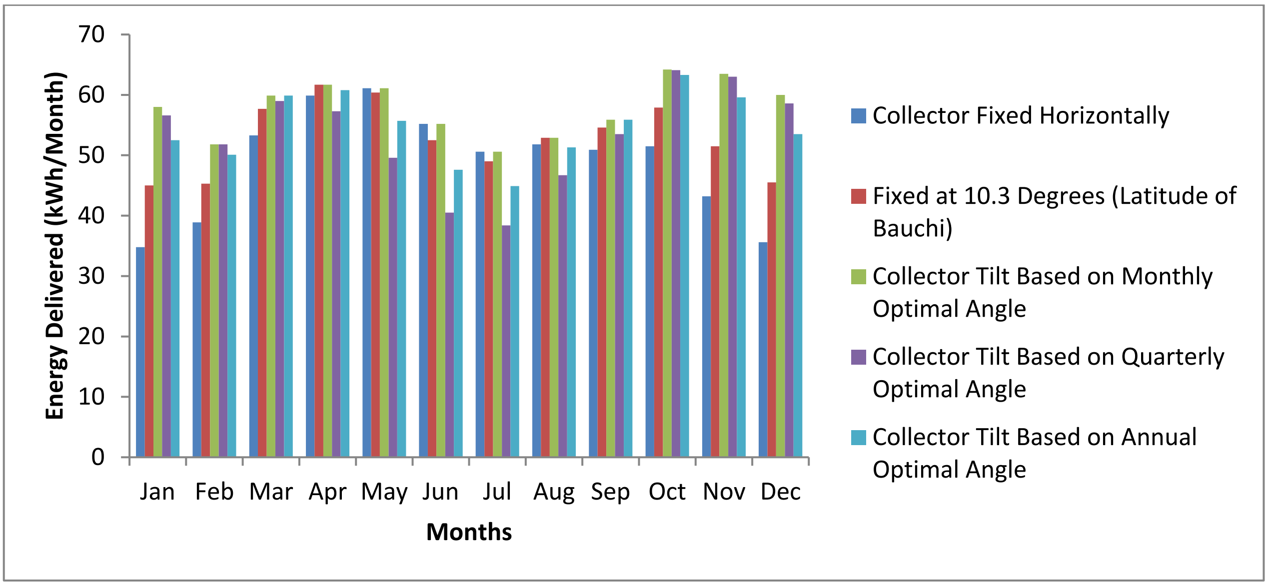

Figure 2: Comparison of energy of the delivered hot water by system based on different mode of collector tilt angle

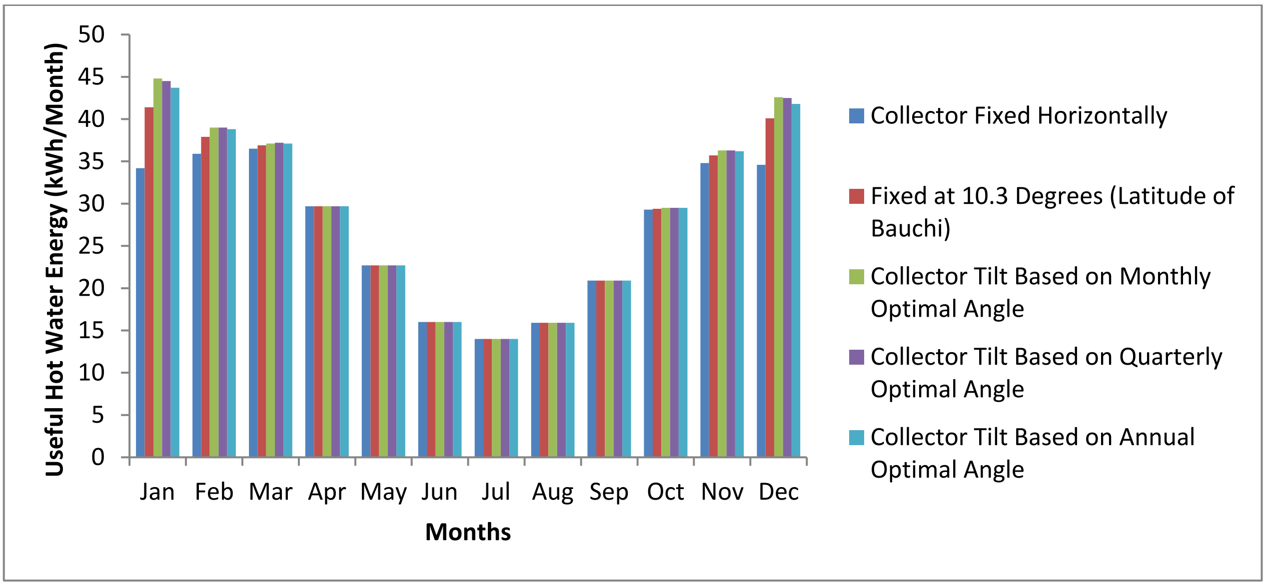

Figure 3: Comparison of useful hot water energy by system based on different mode of collector tilt angle

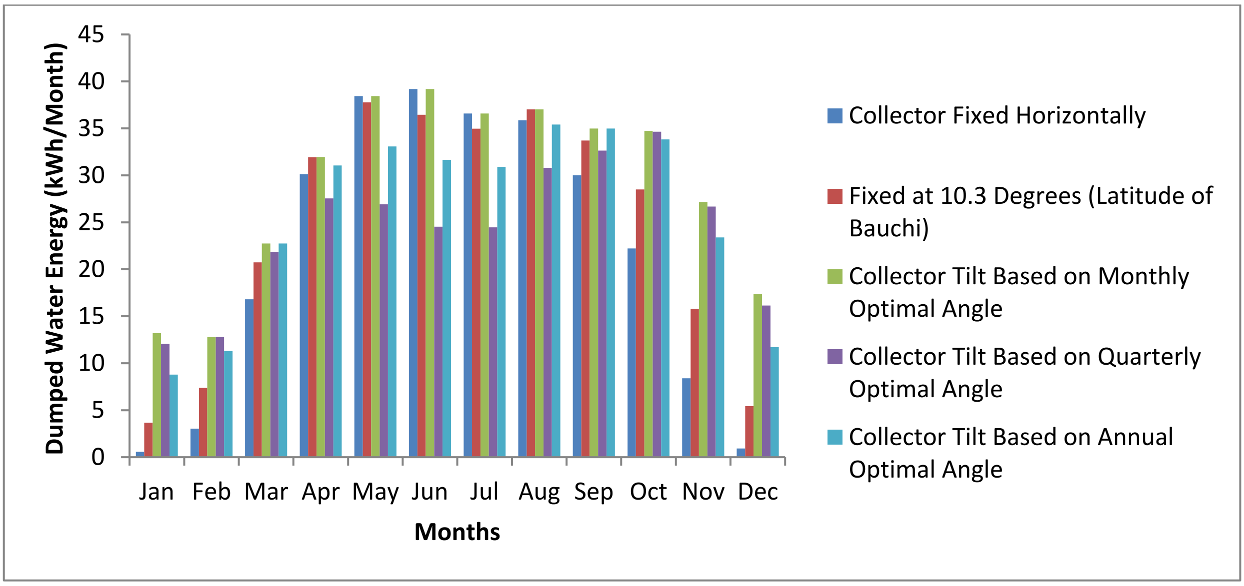

Figure 4: Comparison of dumped (excess) water energy by system based on different mode of collector tilt angle

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Table 2: Advantage of different collector tilt angle relative to the conventional tilt angle

Collector tilt	Performance improvement relative to conventional tilt	Annual energy savings (kWh/year)	Remark
Fixed horizontal	-7	340.6	7% lower in performance than the conventional tilt
Fixed based on monthly optimal	10	324.6	10% higher in performance than the conventional tilt
Fixed based on quarterly optimal angle	1	348.5	1% higher in performance than the conventional tilt
Fixed based on annual optimal angle	3	348.1	3% higher in performance than the conventional tilt

4.0 Conclusion

There is improvement in the amount of solar radiation harnessed by the flat solar collector inclined at monthly, quarterly and, annual tilt. In comparison to a collector fixed at location latitude, the quarterly, annual and, monthly optimal tilt had increment of 1, 3 and 10% respectively for solar energy received. The flat plate water heater had maximum total energy for delivered hot water at monthly tilt angles. This technique of estimating energy harnessed by a flat collector inclined optimally using broaden coverage of sky conditions has demonstrated increased amount of solar radiation received by the flat plate surface and, will no doubt reduced supplementary energy required for satisfying load demand and materials for construction. It is recommended that the proposed collector be validated through development, and experimentation of its prototype with the resultant orientations.

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